Synthetic Control Under Interference: Detecting and Correcting Bias

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SCM in Political Science

SCM emerged as an important tool for analyzing rare political events:

- **Civil wars:** Coercion, governance, and political behavior in civil war. *Journal of Peace Research*, 2024
- **Polarization:** Partisan Enclaves and Information Bazaars: Mapping Selective Exposure to News. *Journal of Politics*, 2022
- **Far Right:** Do Voters Polarize When Radical Parties Enter Parliament? *American Journal of Political Science*, 2019
- **Religion & Politics:** Government Religious Discrimination, Support of Religion, and Societal Violence in Western Democracies. *Comparative Political Studies*, 2024
- **Political Economy:** From Rents to Welfare: Why Are Some Oil-Rich States Generous to Their People? *American Political Science Review*, 2024
- **Regimes:** The Rush to Personalize: Power Concentration after Failed Coups in Dictatorships. *British Journal of Political Science*, 2023
- **Institutional change:** Comparative politics and the synthetic control method. *American Journal of Political Science*, 2015

Causal Inference and Interference

When policies, conflicts, or shocks *spill over* to neighboring regions, do we still have valid donor pools under Synthetic Control?

Outline

What is the Synthetic Control Method (SCM)?

- Enables inference with a small number (or single) treated units;
- Build a synthetic version of the treated unit as a counterfactual weighting unaffected units.
- Potential outcomes for treated unit:
  - Y_{1t}^N: Outcome in absence of intervention (counterfactual).
  - Y_{1t}^I: Outcome under intervention.
- Treatment effect:

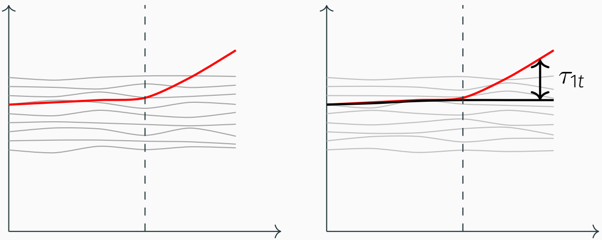
$$\tau_{1t} = Y_{1t}^I - Y_{1t}^N, \quad t > T_0.$$

SCM: How It Works

$$\hat{Y}_{1t}^{N} = \sum_{j=2}^{J+1} w_j Y_{jt}, \quad t > T_0.$$

- Optimal weights W^*: Minimize discrepancy in pre-treatment characteristics and $\|\cdot\|_V$ reflects predictors importance:

$$W^* = \arg\min_{W} \|X_1 - X_0 W\|_V,$$



SCM and SUTVA

- **Stable Unit Treatment Value Assumption (SUTVA):**

$$Y_{it}(Z_i, Z_{-i}) \;=\; Y_{it}(Z_i) \quad \forall i$$

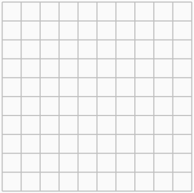
No interference: No unit's outcome depends on other units' treatment status.

- **Crucial Assumption:** The donor units remain *untreated*. Any violation (e.g., partial exposure) can bias the synthetic estimate.
- **SUTVA violation:** Suppose donor j receives an interference term δ_{jt}. The synthetic counterfactual becomes

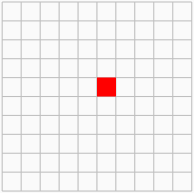
$$\hat{Y}_{it}^{N} \;=\; \sum_{j \neq i} w_j \,(Y_{jt}^{N} + \delta_{jt}),$$

so the estimated effect $\hat{\tau}_{it}$ deviates by $\sum_j w_j \delta_{jt}$ from the *true* τ_{it}.

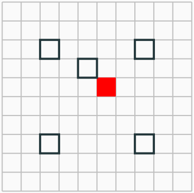
Stages of SCM Construction



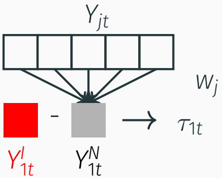
1: Units



2: Single Treated Unit

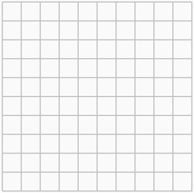


3: Units for Synthetic Control



4: Treatment Effect

Stages of SCM Construction



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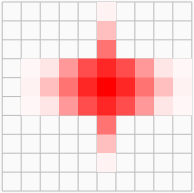
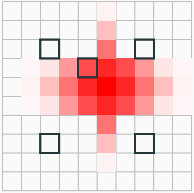


2: Treatment diffusion



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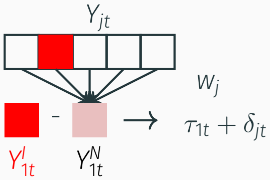
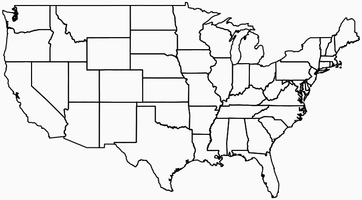


4: Contaminated Treatment Effect

Simulated data



Units map

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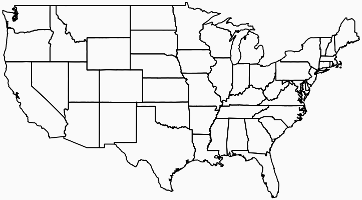

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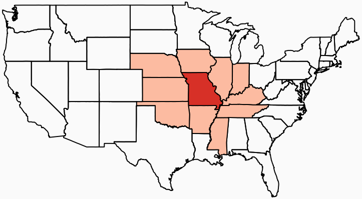

Missouri being treated

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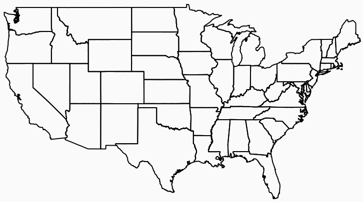

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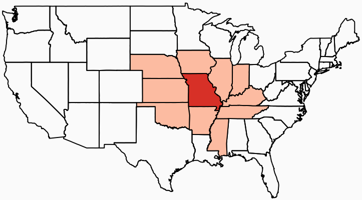
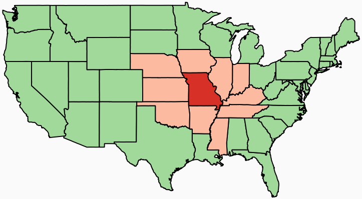


Missouri being treated

Simulated data for an intervention in Missouri with true ATT $\tau = 4$ and interfering the outcome for nearby units by a parameter of $\rho = 0.6$

Closer units are more affected by interference than farther away ones. But how can we compare and test if this interference is at play?

Contrast setup



Contrast for Missouri

Let $i \in \mathcal{U} = \{1, \ldots, N\}$ index units (in this case, US states)

Fix the treated unit ($p \in \mathcal{U}$) at the center and compute distances d_{ip} partitioning the space in non-overlapping rings

$$c_0 < c_1 < \cdots < c_K$$

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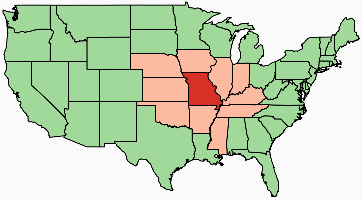

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$$c_0 < c_1 < \cdots < c_K$$

Each ring being identified as:

$$r_{ip} = k \iff c_{k-1} \leq d_{ip} < c_k, \quad k = 1, \ldots, K$$

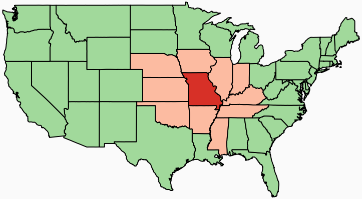
Then assign units to fully disjoint rings according to their distance from p:

- Focus ring: $R_A \subset \{1, \ldots, Q\}$
- Comparison ring: $R_B \subset \{Q+1, \ldots, K\}$

And define groups:

- $A_p = \{i \neq p : r_{ip} \in R_A\}$
- $B_p = \{i \neq p : r_{ip} \in R_B\}$

Contrast setup - Z value



But what are we comparing?

Let $t \in \mathcal{T}$ index time, T_0 be the treatment period for unit p, and Y_{it} represent the outcome

Define two disjoint sets of periods for each window w:

$$\mathcal{T}_w^{\text{pre}}, \mathcal{T}_w^{\text{post}} \subset \mathcal{T}, \quad \mathcal{T}_w^{\text{pre}} \cap \mathcal{T}_w^{\text{post}} = \emptyset$$

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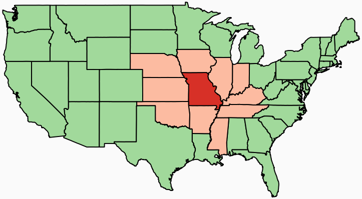

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And set windows of interest for the difference in outcome, such as:

w	$\mathcal{T}_w^{\text{pre}}$	$\mathcal{T}_w^{\text{post}}$
full	$\{t < T_0\}$	$\{t > T_0\}$
year-1	$\{T_0 - 1\}$	$\{T_0 + 1\}$
sym-n	$\{T_0 - n, \dots, T_0 - 1\}$	$\{T_0 + 1, \dots, T_0 + n\}$

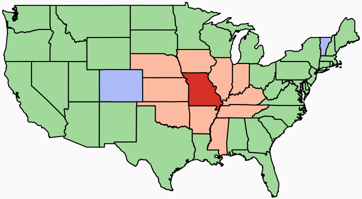
And for every unit i and window w, define a difference-in-means statistic:

$$Z_i^{(w)} = \bar{Y}_{i,\text{post}(w)} - \bar{Y}_{i,\text{pre}(w)}$$

where: $$\bar{Y}_{i,\text{post}(w)} = \frac{1}{|\mathcal{T}_w^{\text{post}}|} \sum_{t \in \mathcal{T}_w^{\text{post}}} Y_{it}$$

and $$\bar{Y}_{i,\text{pre}(w)} = \frac{1}{|\mathcal{T}_w^{\text{pre}}|} \sum_{t \in \mathcal{T}_w^{\text{pre}}} Y_{it}$$

Contrast setup - first test



$Z_i^{(w)} \to$ average outcome variation for each i between post-pre periods in window w.

Anomalous values in units nearby the treated hint at potential interference

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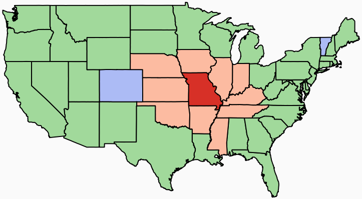

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Anomalous values in units nearby the treated hint at potential interference

state	$Z^{(\text{full})}$	$Z^{(\text{year-1})}$	$Z^{(\text{sym-3})}$
Missouri	4.0066	3.9159	3.9381
Iowa	2.3640	2.4193	2.3539
Colorado	-0.0414	-0.1069	0.0060
Vermont	0.02501	-0.1115	-0.0886

For each window w, collect $Z_i^{(w)}$ for $i \in A_p$ and $Z_i^{(w)}$ for $i \in B_p$, and let

$$\bar{Z}_{A_p}^{(w)} = \frac{1}{|A_p|} \sum_{i \in A_p} Z_i^{(w)}, \quad \bar{Z}_{B_p}^{(w)} = \frac{1}{|B_p|} \sum_{i \in B_p} Z_i^{(w)}$$

denote the group means for each ring set and build:

$$t_p = \frac{\bar{Z}_{A_p} - \bar{Z}_{B_p}}{\sqrt{s_P^2 \left(\frac{1}{|A_p|} + \frac{1}{|B_p|} \right)}}$$

Large $|t_p| \Rightarrow$ evidence that proximity ring(s) differ in mean outcome change relative to farther rings

Contrast setup - randomization

Checking whether average for nearby units	$\neq$	units farther away from treated unit (around treatment)	✓

Can we reject the null of no interference?

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Randomization inference:

$$H_0 : \left\{ Z_i^{(w)} \right\}_{i \in \mathcal{U}} \text{ is invariant to which unit is labelled "treated".}$$

i.e.: Pattern of interference around treated unit is no different than the pattern around any other unit in the space

Contrast setup - randomization II

Algorithm

1. Compute t_p for every $p \in \mathcal{U}$ as above.
2. Let t_0 be the statistic for the **actual treated unit** $p = p^\star$.
3. Exact two-sided p-value:

$$\hat{p} = \frac{1 + \sum_{p \in \mathcal{U}} \mathbf{1}(|t_p| \geq |t_0|)}{N + 1}$$

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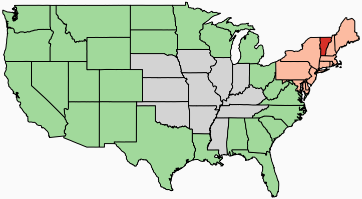

Contrast for Vermont

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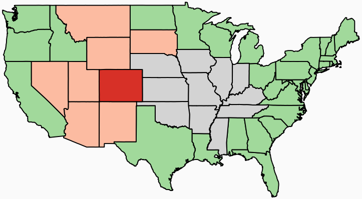
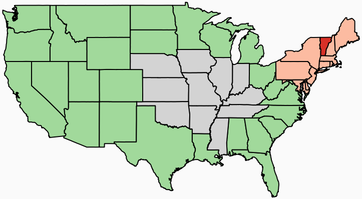


Contrast for Colorado



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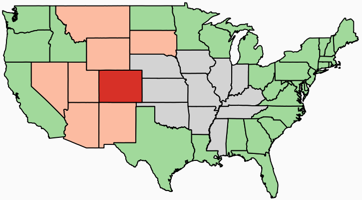

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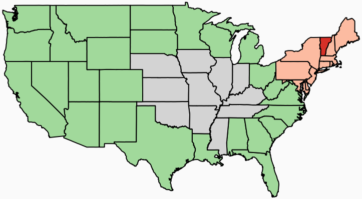
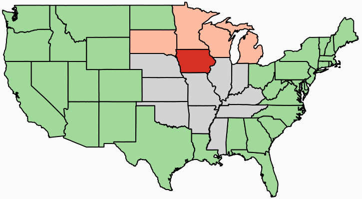


Contrast for Vermont



Contrast for Iowa

Contrast setup- randomization II

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state	t_p	A_p	B_p
MO	4.4207	AR, IL, IN, ...	AL, AZ, CA, ...
VT	-0.2169	CT, DE, ME, ...	AL, AZ, CO, ...
CO	0.3428	AZ, MT, NV, ...	AL, CA, CT, ...
IA	-0.3312	MI, MN, SD, ...	AL, AZ, CA, ...

And from this simulated scenario we obtained p-value = 0.0408

Contrast setup - alternative contrasts

Where does it end?

Detecting whether interference is present ✓

Detecting where interference is no longer statistically significant:

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Detecting where interference is no longer statistically significant:

Instead of contrasting

$A_{p^\star} = \{i \neq p^\star : r_{ip^\star} = 1\}$ vs.

$B_{p^\star} = \{i \neq p^\star : r_{ip^\star} \in \{2, 3, 4, 5\}$

to obtain the standard $t_{p^\star}^{(\text{1 vs 2:5})}$

Contrast: $A_{p^\star} = \{i \neq p^\star : r_{ip^\star} = 2\}$ vs.

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Contrast setup - alternative contrasts

Where does it end?

Detecting whether interference is present ✓

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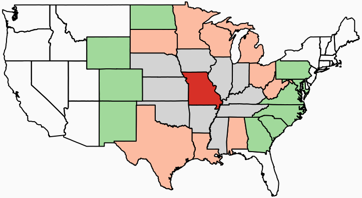
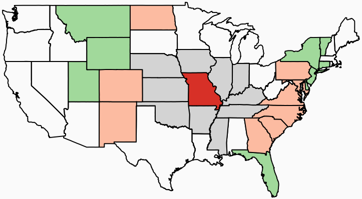


2 vs 3 Contrast for Missouri, p = 0.9591



3 vs 4 Contrast for Colorado, p = 0.5102041

Interference Confirmed. Now What?

Interference ✓

Two options:

- *1. Keeping them unmodified* leads to biased synthetic estimates.
- *2. Simply dropping* suspect donors might degrade the pre-treatment match.

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Interference Confirmed. Now What?

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- *3. Adjust for it:* Use a secondary set of weights to attenuate contamination in the donor pool

    Spatial reach measure as the weights

Spatial Reach: A Continuous Proximity Index

- For donor j, let d_j be its distance to the treated unit.

$$\mathrm{SR}_j = \frac{1}{1 + \exp\left[-\kappa(d_j - c)\right]},$$

  - c is typically the *mean* or *median* distance to center the logistic curve.
  - κ scales how steeply SR_j transitions from near 0 to near 1.

- **Parameter Tuning:** κ trimmed between the 2.5% and 97.5% percentiles of $\{d_j\}$, ensuring a smooth but complete range.
- **Interpretation:** $\mathrm{SR}_j \approx 0$ if donor j is very close, and ≈ 1 if it is far.

Bias Correction Strategies

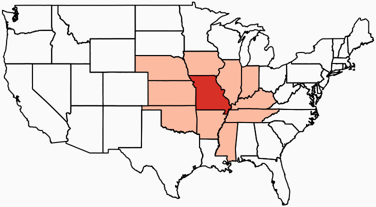
Solution	Optimization	Simplex	Consequence
Rescaling	$\min_w \lVert \mathbf{X}_1 - \mathbf{X}_0^* \, w_j \rVert^2$ with $X^*_{k,j} = X_{k,j} \times \mathrm{SR}_j$	✓	Downweights exposed units; Retains convex weights
Ridge constrained	$\min_w \lVert \mathbf{X}_1 - \mathbf{X}_0 \, w_j \rVert^2 + \lambda \sum_j \mathrm{SR}_j \, w_j^2$	✓	Penalize large SCM weights Moderate contamination
Ridge unconstrained	$\min_w \lVert \mathbf{X}_1 - \mathbf{X}_0 \, w_j \rVert^2 + \lambda \sum_j \mathrm{SR}_j \, w_j^2$	✗	Allows negative SCM weights Aggressively offset contamination

Simplex constraint: $w_j \geq 0, \ \sum_j w_j = 1$

- Units are only allowed to have positive weights
- Unit weights add up to 1

US Simulation

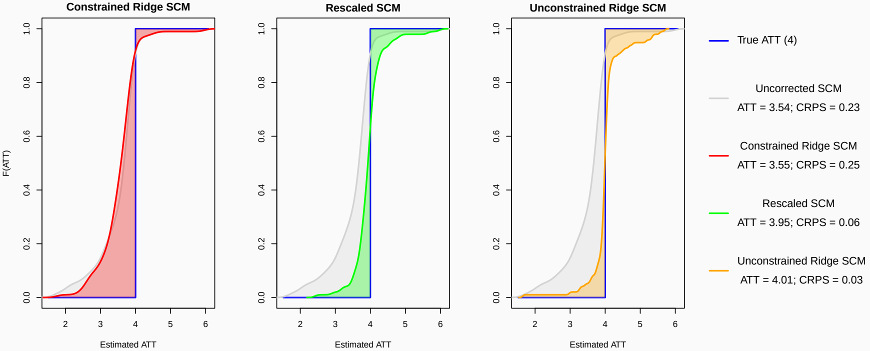
Setup: Intervention in Missouri with true effect size $\tau = 4$ and spillover intensity $\rho = 0.6$.



Compare the uncorrected biased SCM versus the three correction approaches

Metrics: Bias in the estimated ATT, pre-treatment RMSE, and CRPS.

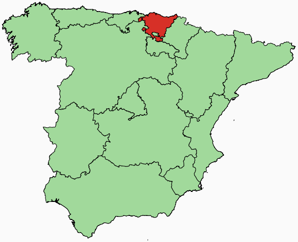
US Simulation results



Simulation under $\tau = 4$ and $\rho = 0.6$

Consistent across all effect sizes τ and spillover intensity ρ

Interference in Applied Research



Abadie et al (2003) Conflict in the Basque
p = 0.22

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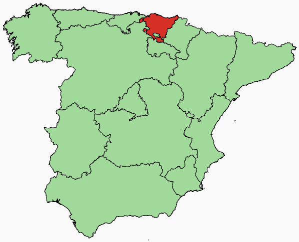

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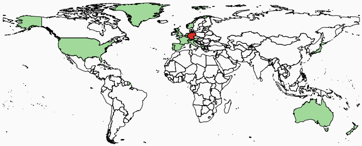

Abadie et al (2015) German Reunification
p = 0.46

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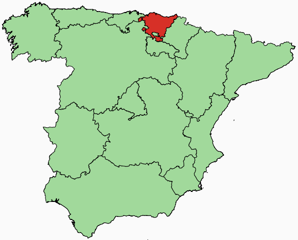

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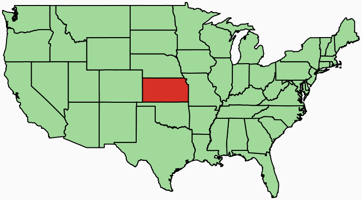
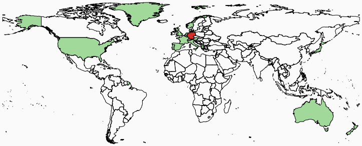
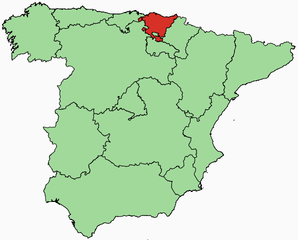


Ben-Michael et al (2021) Kansas tax cut
p = 0.18

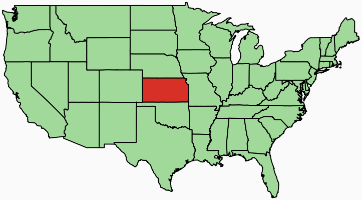


Abadie et al (2015) German Reunification
p = 0.46

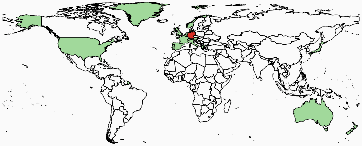
Interference in Applied Research



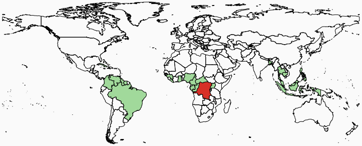
Abadie et al (2003) Conflict in the Basque
p = 0.22



Ben-Michael et al (2021) Kansas tax cut
p = 0.18



Abadie et al (2015) German Reunification
p = 0.46



Kikuta (2020); Civil war and deforestation
p = 0.33

Interference in Applied Research

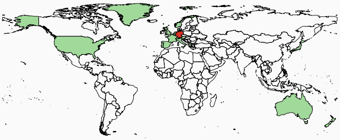
Application	Coverage	Interference
Abadie et al (2003)	✓	✗
Ben-Michael et al (2021)	✓	✗
Abadie et al (2015)	✗	✗
Kikuta (2019)	✗	✗

Interference in Applied Research

Application	Coverage	Interference
Abadie et al (2003)	✓	✗
Ben-Michael et al (2021)	✓	✗
Abadie et al (2015)	✗	✗
Kikuta (2019)	✗	✗
Expanded German Reunification	✓	✓

Interference in Applied Research

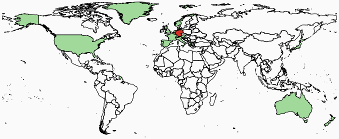
Application	Coverage	Interference
Abadie et al (2003)	✓	✗
Ben-Michael et al (2021)	✓	✗
Abadie et al (2015)	✗	✗
Kikuta (2019)	✗	✗
Expanded German Reunification	✓	✓



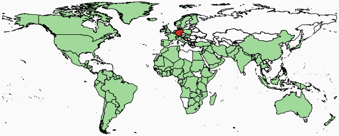
Abadie et al (2015) German Reunification
p = 0.46

Interference in Applied Research

Application	Coverage	Interference
Abadie et al (2003)	✓	✗
Ben-Michael et al (2021)	✓	✗
Abadie et al (2015)	✗	✗
Kikuta (2019)	✗	✗
Expanded German Reunification	✓	✓



Abadie et al (2015) German Reunification
$p = 0.46$



Expanded German Reunification
$p = 0.016$

Interference in Applied Research

Researchers try to address SUTVA violations and patterns of interference by removing units $\rightarrow$ results conditioned on contagion

Risk $\rightarrow$ dropping too many units

Under Potential Outcomes, the DGP and a suitable identification strategy depends on: empirics *AND* how the missing potential outcome is set up

- In the SCM case: which units are in the donor pool

Replication Examples

Comparative politics and the synthetic control method (Abadie, Diamond, & Hainmueller, 2015): German Reunification

Approach	Metric	Germany
Base	ATT Pre-RMSE	-1549.9 119.08
Rescaled	ATT Pre-RMSE	-1601.5 279.03
Penalized, Constrained *	ATT Pre-RMSE	-1103.4 80.43
Penalized, Unconstrained *	ATT Pre-RMSE	136.1 59.5

Rescaling adjusted for contamination → larger effect

Constrained Ridge adjust for contamination and large weights → attenuation

Unconstrained Ridge extrapolate simplex for aggressive correction → reversal

Concluding remarks - Detection

A) Detection

- **Coverage**: Ensure proper donor units coverage to compose the missing potential outcome;
- **Detection test**: Using randomization inference, assess whether interference is at place in the empirical setting;
- **Alternative contrast**: By adapting the contrast, identify where interference is no longer detected;
- **Detect Interference First**: If no violation is detected, standard SCM suffices;

Concluding remarks - Correction

B) Correction

- **SR weight**: If interference $\rightarrow$ subject the SCM optimization problem to network-specific weights;
- **Minor to moderate interference**: Rescaling or Constrained Ridge can mitigate moderate bias while retaining the notion of a convex combination.;
- **Severe Interference**: Unconstrained Ridge achieves lower bias at the cost of extrapolating out of the simplex;

Ongoing Extensions

- **Inverse Propensity Weighting for Rescaling Approach**

  HT–Hájek Spatial Weights

  Spatial–reach $f(d)$ as *propensity to avoid spillover*: $\pi_i = 1 - f(d_{iD})$

  Use stabilized Horvitz–Thompson weights $w_i = \dfrac{1/\pi_i}{\sum_j 1/\pi_j}$ inside SCM

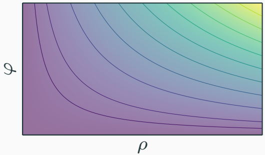
- **Multiple Comparison & Dynamic Networks**

- **Sensitivity to Interference**

  Inject controlled spillovers in outcomes & covariates: intensity $\rho \in [0, 1]$, decay φ

  Re-run SCM over a (ρ, φ) grid; track standardized shift

  Contours show ATT shift required to overturn conclusions



(Lighter $\rightarrow$ larger ATT shift)